

Below is a list of all of our series convergence/divergence test

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① Divergence Test: If  $\lim_{n \rightarrow \infty} a_n \neq 0$ , then  $\sum a_n$  diverges

- Use if  $\lim_{n \rightarrow \infty} a_n$  is easy enough to calculate
- Deals w/ series that are "obviously" divergent

② p-Series Test: A series of the form  $\sum \frac{1}{n^p}$  converges if  $p > 1$  and diverges if  $p \leq 1$ .

- Use if specifically in the form  $\sum \frac{1}{n^p}$

③ Geometric Series Test: A series of the form  $\sum a \cdot r^n$  converges if  $|r| < 1$  and diverges if  $|r| \geq 1$ .  
If  $|r| < 1$ ,  $\sum a \cdot r^n$  converges to  $\frac{\text{(first term)}}{1-r}$ .

- Use if specifically in the form  $\sum a \cdot r^n$
- Our only test that we can determine what value a series converges to.

④ Alternating Series Test (AST): Consider an alternating series  $\sum (-1)^n a_n$  or  $\sum (-1)^{n+1} a_n$ . If  $\{a_n\}$  positive

- $\lim_{n \rightarrow \infty} a_n = 0$

and

- $\{a_n\}$  is decreasing,

then the alternating series is convergent.

- Consider if there is an  $(-1)^n$  or  $(-1)^{n+1}$ .
- If  $\lim_{n \rightarrow \infty} a_n \neq 0$ , we could use Divergence Test.

⑤ Direct Comparison Test (DCT): Let  $a_n < b_n$  be positive sequences,

- If  $\sum b_n$  is convergent, then  $\sum a_n$  is also convergent.

• If  $\sum a_n$  is divergent, then  $\sum b_n$  is also divergent.

• Use if the series resembles a p-series or geometric series.

⑥ Limit Comparison Test (LCT): Let  $a_n, b_n$  be positive sequences

• If  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} > 0$  and finite, then  $\sum a_n$  and  $\sum b_n$  both converge or both diverge.

• Use if the series resembles a p-series or geometric series.

⑦ Ratio Test: Consider  $\sum a_n$  and let  $L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$ .

• If  $L < 1$ ,  $\sum a_n$  converges (absolutely)

• If  $L > 1$ ,  $\sum a_n$  diverges

• If  $L = 1$ , the Ratio Test is inconclusive.

• Use if there are factorials

• Also works well if there are exponentials

⑧ Root Test: Consider  $\sum a_n$  and let  $L = \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$

• If  $L < 1$ ,  $\sum a_n$  converges (absolutely)

• If  $L > 1$ ,  $\sum a_n$  diverges

• If  $L = 1$ , the Root Test is inconclusive

• Use if  $\sum (a_n)^n$

⑨ Absolute Convergence Test: If  $\sum |a_n|$  converges, then  $\sum a_n$  converges (absolutely).

• Use if there are negative terms (not necessarily alternating)

⑩ Integral Test: If  $a_n = f(n)$  is positive and decreasing, then  $\int_k^{\infty} f(x) dx$  and  $\sum_{n=k}^{\infty} a_n$  both converge or diverge.

• Use if antiderivative is easy enough to find

Note: The tests at the top of the list should be used more often than those at the bottom.



