MATH 1132Q – Exam 2 Practice (Fall ’24) – Lucien Piazza

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Note:

- Time yourself based on time given for actual exam
- Remember to take breaks between long problems and recollect your thoughts
- Skip around the exam as needed, all the problems are weighed pretty much the same
- My recommended order for solving the problems (first to last):
    - P1, P7, P2, P10, P6, P5, P3, P4, P9, P8
    - Also, you can follow Prof. Ben Lantz’s Guide to Series Convergence Tests, which ranks the order of each test from Easiest (solve first) to Hardest (solve last)

Note: Each problem has a # next to it. This is my recommended order in which to solve the problems!

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MATH 1132 EXAM 2 PRACTICE FALL 2024

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Read This First!

- Please read each question carefully. The point value of each question is indicated in the box preceding its statement.
- In all free-response questions, you will *only* receive credit by providing clear work using techniques learned in this class.
- No books, formula sheets, calculators, phones, smart watches, headphones, or other references are permitted.
- No personal scrap paper is allowed.

Practice Problems Notes

- The practice problems together are almost twice as long as the actual exam.
- Timed practice without notes is essential! Try some of these under exam conditions after you have had some time to study.
- The coverage in the practice problems is not exhaustive. Look through class notes, WebAssign, and quizzes as well.

(First term)

$n=1$

1

Do these first. No worries if you have to skip a few to come back to later!

Problem 1: Indicate the validity of each statement by entering True (T) or False (F) in the blank.

FIRST: Is $|r|<1$? → If yes, it is convergent! Conv.s to $\frac{2\left(\frac{1}{3}\right)^{1-1}}{1-1/3} = \frac{2}{2/3} = \boxed{3}$ ✓

(i) T The geometric series $\sum_{n=1}^{\infty} 2\left(\frac{1}{3}\right)^{n-1}$ converges to 3.

$\sum_{n=\#}^{\infty} a(r)^k = \frac{a}{1-r}$ → What value series conv.s to

a → First term in series

(ii) F If the series $\sum_{n=1}^{\infty} a_n$ diverges, then the sequence a_n must diverge.

Sequence → List/pattern of #s ; Ex. $a_n = \frac{1}{n}$: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \approx \emptyset$

Series → Sum of a sequence ; Ex. $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ → Diverges!

Ex. Harmonic series

(iii) T The sequence $a_n = \frac{n^2}{e^n}$ has limit 0.

A limit of $\emptyset$ means that denominator must dominate → e^n dom.s n^2 since exponentials dom. quadratics!

L'Hôpital's can be used twice to solve too! $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f''(x)}{g''(x)}$

(iv) T The sequence $a_n = (-1)^n$ is bounded.

Alternating series bounded by -1 and 1

$S_1 = -1,\ S_2 = 0,\ S_3 = -1,\ S_4 = 0, \ldots$ → NOT convergent! (Divergent)

(v) T The partial sums of the series $\sum_{n=0}^{\infty} \frac{1}{2^n}$ are an increasing sequence.

$S_n = \underset{a_1}{1},\ \underset{a_1}{1} + \underset{a_2}{\frac{1}{2}},\ \underset{a_1}{1} + \underset{a_2}{\frac{1}{2}} + \underset{a_3}{\frac{1}{4}}, \ldots$ → Adding positive terms

(vi) T If we use the partial sum S_5 to approximate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$, the Alternating Series Remainder Estimate tells us that the remainder/error satisfies $|R_5| \le \frac{1}{7}$.

$a_n = \frac{1}{n+1}$ $|R_n| \le a_{n+1}$, so $|R_5| \le \frac{1}{(5+1)+1} = \frac{1}{7}$ ✓

(vii) F If $b_n > a_n > 0$ for all n and $\sum b_n$ diverges, then $\sum a_n$ must converge.

→ If this said conv.ed then true!

b_n ↓ Div. a_n ↓ Conv.

This statement is inconclusive since $a_n < b_n$ (tells us nothing about whether or not b_n is finite)

(viii) F The series $\sum_{n=1}^{\infty} \frac{3n-1}{5n+2}$ converges.

$\lim_{n\to\infty} \frac{3n-1}{5n+2}$ (1 → Ignore, 2 → Ignore) → $\lim_{n\to\infty} \frac{3\cancel{n}}{5\cancel{n}} = \frac{3}{5} \neq 0$ → Diverges by Divergence Test

Next, look for & do all Ratio Test problems since they are easy to spot

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Problem 2: Determine whether $\sum_{n=0}^{\infty} \frac{n^5}{n!}$ converges or diverges. Please show all work and clearly indicate any test being used. → Factorial → Ratio Test!

Ratio Test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\frac{(n+1)^5}{(n+1)!}}{\frac{n^5}{n!}}\right| = \lim_{n\to\infty} \frac{(n+1)^5}{(n+1)!} \cdot \frac{n!}{n^5}$

$= \lim_{n\to\infty} \frac{(n+1)^5}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{n^5}$

$= \lim_{n\to\infty} \frac{(n+1)^5}{n^5(n+1)} \sim \frac{n^5}{n^5 \cdot n}$ Denom. dominates

$= \emptyset$

Since $\lim_{n\to\infty} \frac{(n+1)^5}{n^5(n+1)} = \emptyset$, $\sum_{n=0}^{\infty} \frac{n^5}{n!}$ converges by the Ratio Test.

7 Medium-level LCT

Problem 3: Determine whether $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+5n}}{n^3}$ converges or diverges. Please show all work and clearly indicate any test being used. → Manipulate

a_n resembles $\sum_{n=1}^{\infty} \frac{\sqrt{n^2}}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ → Converges by p-series since $p = 2 > 1$.

LCT: $\lim_{n\to\infty} \frac{\frac{\sqrt{n^2+5n}}{n^3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+5n}}{n^3} \cdot \frac{n^2}{1} = \lim_{n\to\infty} \frac{\sqrt{n^2+5n}}{n}$

$= \lim_{n\to\infty} \frac{\sqrt{n^2\left(1+\frac{5n}{n^2}\right)}}{n}$

$= \lim_{n\to\infty} \frac{\cancel{n}\sqrt{1+\frac{5n}{n^2}}}{\cancel{n}}$

$= \lim_{n\to\infty} \sqrt{1+\frac{5\cancel{n}}{\cancel{n^2}}} = \sqrt{1} = 1$

Since $\lim_{n\to\infty} \frac{\frac{\sqrt{n^2+5n}}{n^3}}{\frac{1}{n^2}} = 1$ & $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series ($p = 2 > 1$), $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+5n}}{n^3}$ must also converge by LCT.

Note: D.C.T. does not work here since inequality doesn't!

I.T., longer & annoying but this one is doable

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★ Quotient rule, low d-high, high d-low over $(\text{low})^2$ ★

Problem 4: Use the Integral Test (and check all conditions) to determine whether $\sum_{n=3}^{\infty} \frac{\ln(n)}{n}$ converges or diverges. Please show all work.

FIRST check conditions:

- Positive → $n > 0$ & if $n \geq 3$, $\frac{\ln(n)}{n} > 0$ for all n ✓
- Decreasing → $f'(n) = \frac{n \cdot \frac{1}{n} - \ln(n) \cdot 1}{n^2} = \frac{1 - \ln(n)}{n^2}$; If $n \geq 3$, $\ln(n) > 1$ & is inc.ing. Thus $f'(n) < 0$ & $\frac{\ln(n)}{n}$ is decreasing. ✓

Integral Test: u-sub, $u = \ln(x)$

$$\lim_{b\to\infty} \int_3^b \frac{\ln(x)}{x}\,dx = \lim_{b\to\infty} \frac{1}{2}(\ln(x))^2 \Big|_3^b = \lim_{b\to\infty} \Big(\underbrace{\tfrac{1}{2}(\ln(b))^2}_{\infty} - \underbrace{\tfrac{1}{2}(\ln(3))^2}_{\text{Constant, insignificant for } b\to\infty} \Big) = \infty$$

so $\int_3^\infty \frac{\ln(x)}{x}\,dx$ diverges.

Thus, $\sum_{n=3}^{\infty} \frac{\ln(n)}{n}$ also div.s by the Integral Test.

Requires some thought for setup but super quick & easy

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Problem 5: What is the smallest value of N for which the Alternating Series Remainder Estimate tells us the remainder R_N for the Nth partial sum of $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ satisfies $|R_N| \leq \frac{1}{10}$? Please show all work.

$$= \sum_{n=1}^{\infty} (-1)^n \cdot \underbrace{\frac{1}{\sqrt{n}}}_{a_n}$$

A.S.R.E.:

$$|R_n| \leq a_{n+1} \to |R_n| \leq \frac{1}{\sqrt{n+1}} \left(\leq \frac{1}{10} \right)$$

We want N such that $\frac{1}{\sqrt{n+1}} \leq \frac{1}{10}$ → Solve for N

$$\sqrt{N+1} \leq 10 \to (\sqrt{N+1})^2 \leq 10^2 \to N+1 \leq 100 \to N \leq 99$$

$$\boxed{\text{Smallest value of } N = 99}$$

Easy comparison test

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Problem 6: Determine whether $\sum_{n=2}^{\infty} \frac{n}{n^2-3}$ converges or diverges. Please show all work and clearly indicate any test being used.

Resembles $\sum_{n=2}^{\infty} \frac{n}{n^2} = \sum_{n=2}^{\infty} \frac{1}{n} \rightarrow$ This is the Harmonic Series, which diverges by p-series ($p=1\leq 1$)

D.C.T.: $\underbrace{\frac{n}{n^2-3}}_{b_n} > \underbrace{\frac{n}{n^2}}_{a_n} = \frac{1}{n} \rightarrow \frac{n}{n^2-3}$ is larger since its denom. contains smaller value, resulting in a larger #.

$\rightarrow$ Smaller series diverges, therefore larger series, b_n, diverges as well.

$$\boxed{\text{Since } \frac{n}{n^2-3} > \frac{1}{n} \text{ \& } \sum_{n=2}^{\infty}\frac{1}{n} \text{ diverges by p-series } (p=1\leq 1),\ \sum_{n=2}^{\infty}\frac{n}{n^2-3} \text{ diverges by D.C.T.}}$$

Always scan through & look for easy D.T. problems right away!

Note: D.C.T. is quicker but L.C.T. works perfectly fine too!

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Problem 7: Determine whether $\sum_{n=0}^{\infty} \frac{n!}{n^2+6^n}$ converges or diverges. Please show all work and clearly indicate any test being used.

$\rightarrow$ Factorial in numerator! $\rightarrow$ D.T.

Divergence Test:

factorial polynomial exponential

$\lim_{n\to\infty} \frac{n!}{n^2+6^n} = \infty$ since $n!$ dominates n^2 & 6^n.

Therefore, $\sum_{n=0}^{\infty} \frac{n!}{n^2+6^n}$ diverges by the Divergence Test

Note: Ratio Test could be used but D.T. is much quicker

Longest problem, worth same amount of pt.s as others

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Problem 8: Determine whether the series

$$\sum_{n=2}^{\infty}(-1)^n \frac{n}{\sqrt{n^4-1}}$$

is absolutely convergent, conditionally convergent, or divergent. Please show all work and clearly indicate any test being used.

1. Check $\sum a_n$: A.S.T. w/ $a_n = \frac{n}{\sqrt{n^4-1}}$

- $\lim_{n\to\infty} \frac{n}{\sqrt{n^4-1}} \sim \lim_{n\to\infty} \frac{n}{\sqrt{n^4}} = \lim_{n\to\infty} \frac{n}{n^2} = \emptyset$
- $f'(n) = \frac{\sqrt{n^4-1}\cdot 1 - n\left(\frac{4n^3}{\sqrt{n^4-1}}\right)}{n^4-1} = \frac{\frac{n^4-1-4n^4}{\sqrt{n^4-1}}}{n^4-1} < \emptyset$

$n^4-1-4n^4 < \emptyset$, so

Since $f'(n)<0$, a_n is dec.ing. Thus, $\sum_{n=2}^{\infty}(-1)^n \frac{n}{\sqrt{n^4-1}}$ conv.s by A.S.T.

2. Check $\sum |a_n| = \sum_{n=2}^{\infty} \frac{n}{\sqrt{n^4-1}} \sim \sum_{n=2}^{\infty} \frac{n}{n^2} \rightarrow$ Compare to $\sum_{n=2}^{\infty} \frac{1}{n}$ which div. by p-series. $(p=1\le 1)$

D.C.T.:

$\frac{n}{\sqrt{n^4-1}} > \frac{n}{\sqrt{n^4}} = \frac{n}{n^2} = \frac{1}{n}$

Since $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges & $\frac{n}{\sqrt{n^4-1}} > \frac{1}{n}$, $\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^4-1}}$ diverges by D.C.T.

Thus, $\sum_{n=2}^{\infty}(-1)^n \frac{n}{\sqrt{n^4-1}}$ is conditionally convergent.

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Problem 9: Determine whether the series

Longer ACT but quick if you start with $\sum |a_n|$

$$\sum_{n=2}^{\infty}(-1)^n \frac{n^3}{n^5+6}$$

is absolutely convergent, conditionally convergent, or divergent. Please show all work and clearly indicate any test being used.

1. Check $\sum |a_n| = \sum_{n=2}^{\infty} \frac{n^3}{n^5+6}$ first: $\sum_{n=2}^{\infty} |a_n| \sim \sum_{n=2}^{\infty} \frac{n^3}{n^5} = \sum_{n=2}^{\infty} \frac{1}{n^2} \rightarrow$ Conv. by p-series $(p=2>1)$

L.C.T.: $\lim_{n\to\infty} \frac{\frac{n^3}{n^5+6}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^5}{n^5+6} = 1 \rightarrow$

Since $\sum_{n=2} \frac{1}{n^2}$ converges by p-series $(p=2>1)$, $\sum_{n=2}^{\infty} \frac{n^3}{n^5+6}$ also conv.s by L.C.T.

Since $\sum_{n=2}^{\infty} |a_n|$ converges, $\sum_{n=2}^{\infty} (-1)^n \frac{n^3}{n^5+6}$ converges absolutely by A.C.T.

Another Ratio Test!

4 **Problem 10:** Determine whether $\sum_{n=1}^{\infty} \frac{n^2 7^n}{n!}$ converges or diverges. Please show all work and clearly indicate any test being used.

Ratio test

Ratio Test:

split up

Group polynomials together

Group exp.s together

Group facts together

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{(n+1)^2 7^{n+1}}{(n+1)!}}{\frac{n^2 7^n}{n!}} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^2}{n^2} \cdot \frac{\cancel{7^n} \cdot 7}{\cancel{7^n}} \cdot \frac{\cancel{n!}}{(n+1)\cancel{n!}} \right|$$

Numerator: $\sim 7n^2$

$$= \lim_{n\to\infty} \left| \frac{7(n+1)^2}{n^2(n+1)} \right| = \varnothing$$

Denominator: $\sim n^3$

$\boxed{\text{Therefore, } \sum_{n=1}^{\infty} \frac{n^2 7^n}{n!} \text{ converges by the Ratio Test.}}$