University of Connecticut
Department of Mathematics

MATH 1132 EXAM 1 PRACTICE FALL 2024

NAME: ______________________________

SIGNATURE: ______________________________

TA Name: ______________________________ Class Time: __________

Read This First!

- Please read each question carefully. The point value of each question is indicated in the box preceding its statement.
- In all *short answer* questions, you will *only* receive credit by providing clear work using techniques learned in this class.
- No books, formula sheets (other than the one provided), calculators, smart watches, or other references are permitted.
- No personal scrap paper is allowed. If you need additional space to write, it will be provided by your instructor. All scrap paper must be turned in with the exam.

Notes about the Practice Exam

- This practice exam is about twice as long as the actual exam.
- Please use this as a practice exam and try each problem before jumping to the solutions.
- The format of the exam will be similar to this practice exam, but you should not expect the same problems (or even the same kinds of problems).
- Using this as a form of studying is encouraged, but you are expected to study beyond this as well using your notes, homework, extra practice problems, office hours, study groups, etc.

Formula Sheet

Trigonometric identities.

$$\sin^2\theta + \cos^2\theta = 1 \qquad \sin 2\theta = 2\sin\theta\cos\theta \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$1 + \tan^2\theta = \sec^2\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Approximate Integration Formulas.

Midpoint Rule. $\boxed{M_n = \Delta x(f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n))}$, where $\overline{x}_i = \frac{1}{2}(x_{i-1} + x_i)$

The error bound in the Midpoint Rule is $|E_M| \le \dfrac{K(b-a)^3}{24n^2}$, where K is chosen so that $|f''(x)| \le K$ for $a \le x \le b$.

Trapezoidal Rule. $\boxed{T_n = \dfrac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n))}$

The error bound in the Trapezoidal Rule is $|E_T| \le \dfrac{K(b-a)^3}{12n^2}$, where K is chosen so that $|f''(x)| \le K$ for $a \le x \le b$.

Simpson's Rule (For *even* n).

$$\boxed{S_n = \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))}$$

The error bound in Simpson's Rule is $|E_S| \le \dfrac{K(b-a)^5}{180n^4}$, where K is chosen so that $|f^{(4)}(x)| \le K$ for $a \le x \le b$.

Problem 1: When using a partial fraction decomposition to compute $\displaystyle\int \frac{7x+11}{(x-1)(x+1)(x+2)}\,dx$, we would write $\displaystyle\frac{7x+11}{(x-1)(x+1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$. Find A, B, and C.

Problem 2: What is the correct form of the partial fraction decomposition that should be used in order to evaluate the integral $\displaystyle\int \frac{3x^2+1}{x^3+16x}\,dx$?

Problem 3: Evaluate $\int (x^2 + 5)\sin(2x)\, dx$

Problem 4: Evaluate $\int \arctan(x)\, dx$

Problem 5: Compute the value of $\int_2^5 \frac{1}{\sqrt{x-2}}\,dx$ or say that it diverges.

Problem 6: Compute the value of $\int_{-\infty}^1 e^{5x}\,dx$ or say that it diverges.

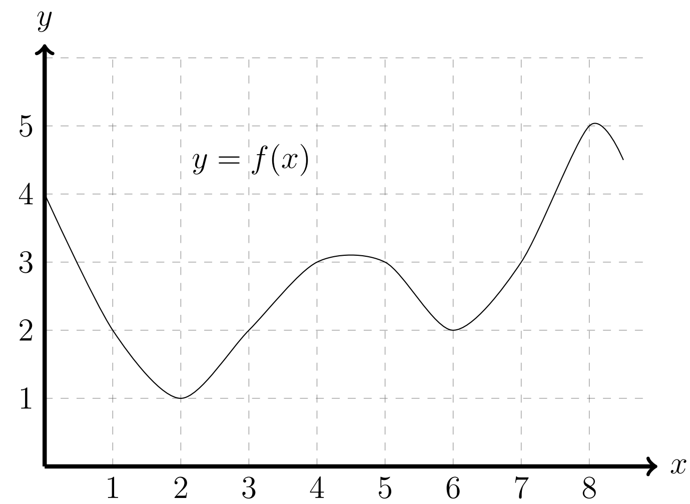
Problem 7: If $x = 5\tan(\theta)$ then describe $\theta + \sec(\theta)$ in terms of x.

Problem 8: Evaluate $\displaystyle\int \frac{1}{\sqrt{25 - 9x^2}}\,dx$ using a *trigonometric substitution.*

Problem 9: Evaluate $\int \sec^4(x)\tan^2(x)\,dx$.

Problem 10: Evaluate $\int \cos^4(x)\,dx$.

Problem 11: Using the graph of $f(x)$ below, find T_4 (the *trapezoidal approximation* with $n = 4$) for $\int_0^8 f(x)\,dx$. You do *not* need to simplify your answer, but it should consist of only numbers (there should not be any function notation).



Problem 12: Consider the integral $\int_1^3 \sqrt{x}\,dx$. *Show all of your work to receive credit.*

(A) Express the Midpoint Rule approximation M_3 for this integral as a sum of numbers. You do *not* need to simplify your answer.

(B) What is the smallest value of K we can use when computing the error bound for the Midpoint Rule.

(C) What is the smallest value of n for which the error bound from estimating the integral by M_n is at most $\frac{1}{1200}$