

Chapter 14 Review – Partial Derivatives

Hello everyone, it's Akeva!

I know you all have an exam coming up on with partial derivatives, so this document is supposed to serve as a guide to the general types of problems you may encounter and how to approach each type of problem! I will go over definitions when needed but be sure to go through your textbook and/or worksheets for any additional information you may need. Also, feel free to refer to this document to help generate your cheat sheet. I try to cater to many learning types (reading (my blurbs), visual (videos/going through problems), listening (videos), and kinesthetic (walking through problems)). If you like this format, let me know!

How I would study if I redid Multi

Since this is technically my first piece of content for y'all I want to break down how I would approach studying for Multi exams (note: I usually study for exams 2-3 days in advance, hence this list):

1. Redo any homework problems or quiz questions you had a hard time answering
2. Go through practice exam questions given without any notes
3. When going through the exam questions again, mark what you go wrong like a professor or TA and simultaneously make your cheat sheet with things that could have helped you solve that problem better (making your cheat sheet is part of your studying)
 - a. When making your cheat sheet, if just equations suffice for you, just make it equations. However, for me, I would often put steps and signals for when I should use a certain technique vs another as that used to be a weak point of mine
4. Try doing additional practice problems on ones you have not seen before such as YouTube videos (refer to my list down below) and use your cheat sheet
 - a. Grade your cheat sheet as it goes. How much is it helping you? What is missing (make sure you add it)?
5. Try to get at least 5-6 hours of sleep
 - a. I'm being realistic for me here. I was not the student who made sure I got 8 hours of sleep (even on a good day, haha).
6. Don't freak out about not understanding a problem or two. Stress can psyche you out.

Basics

Equation Recognition

Alright, let's kinda do basics here. I think an integral (no pun intended) part of understanding these problems are recognizing common shapes by their equations and drawing them. I know it may seem annoying, but you are sometimes tested on this, and I think it can help ground you in

the problem and better understand what is being asked. So as a refresher, you should know the following equations and their general shapes based on recognition:

- Sphere/Ellipsoid:
 - $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
 - *Note that when a, b, and c all equal the same value (doesn't need to be 1), the shape is a sphere*
- Cone:
 - $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$
- Elliptic paraboloid:
 - $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
- Hyperbolic paraboloid:
 - $z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$

Based on these equations, you should also be able to determine the 2D version on these shapes. For example, if I were to ask you what the following shapes was, you should look at it and say what:

$$x^2 + y^2 = 16$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

... A circle and ellipse, respectively. If you are unsure, you could also quickly graph the points where $x = 0$ and $y = 0$ (hint hint, you should get two values due to you ultimately taking the square root. Ex: $x^2 + y^2 = 16$ turns into $y^2 = 16$ when $x = 0$ and then $y = \sqrt{16} = \pm 4$, yielding the points (0, 4) and (0, -4). Doing the same for x, you should also get (4, 0) and (-4, 0). At this point, you would just draw curved lines between the points because we mainly work with shapes that have curves to them. Based on equidistance between these points in this problem, we can conclude that this is a circle. If you found graphed this by hand, try doing the same for the second equation, going step by step.

Definition time (for the next chunk of text)!

I like to think of level curves as these of the mini 2D shapes that make up an overall 3D shape. If you are familiar with 3D printing or drawing, you know that you do things in layers of shapes; it's the same thing here.

Problem: Based on this, if I were to give you tell you sketch the level curves of the function $g(x, y) = \sqrt{9 - x^2 - y^2}$ for $k = 0, 1, 2,$ and 3 , you should recognize some key things:

- Although the format of it is a little different, it is a circle. How can I tell? Well, I noticed that both the square of x and y are negative. Meaning, if I set $9 - x^2 - y^2 = 0$ and then

move the square of x and y to the other side of the equation $9 = x^2 + y^2$, we get a familiar looking equation.

- Level curves are often done in 2D, so we should get multiple circles on our graph that are varying in radii. But how do we determine the radii? Awesome question.

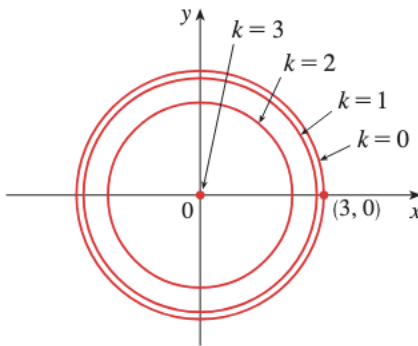
You should just set the equation given to you equal to a variable. The problem already defines it for us a k, so we do $\sqrt{9 - x^2 - y^2} = k$. Square roots are ugly to work with so let's take the square of both sides (remember we need to do this so that we keep everything equal to before).

$$\begin{aligned}\sqrt{9 - x^2 - y^2} &= k \\ 9 - x^2 - y^2 &= k^2\end{aligned}$$

Let's make the equation like that of a traditional circle equation with positive squares of x and y on the same side, leaving the other side to be the radius squared.

$$9 - k^2 = x^2 + y^2$$

So now we know that the radius is $\sqrt{9 - k^2}$. Then plugging in 0, 1, 2, and 3 into the k, we find circles with a radius of 3, $\sqrt{8}$, $\sqrt{5}$, and 0 (aka a point). Graphing everything and putting it together, you should get a graph very similar to the following:



Derivatives and Integrals

For this section, I would suggest that go over the following [cheat sheet](#) that goes over very important topics that I will go over in the coming sections such as:

- Chain rule
- Product Rule
- Quotient Rule
- Harder integrals or derivatives such as $\ln(x)$ or $\cos(x)$

I mainly leave this section up to you as this should be review. However, if there are some things you do not understand regarding derivatives and integrals, refer to the following videos :) :

- [Derivatives](#)
- [Integrals](#)

If after going through everything and you're still confused about derivatives and integrals, feel free to come find me or another Math specialist and we'll be glad to help! :)

Unit Circle

Refer to [this video](#) by Professor Dave Explains. I think he does a good job of explaining the topics in a way that is not daunting. This will be helpful in case you encounter problems with in Cylindrical and Spherical coordinates.

Chapter 14

Finding Domains of Functions

If you're like me and not too fond on reading, I found a video that goes through the initial textbook problems in section 14.1 by [CBlissMath](#). I think she explains these problems well and how you should be thinking about finding the domain. Don't worry about difference in notation, notably the curly braces. Just focus on knowing the domain and during exams, you can show the domain through words or more effectively through graphs as she does in this video.

Another example that is not necessarily covered by the textbook is in the following video by [Krista King](#)! This also test where you are in terms of recognizing equations. As the videos go over, don't worry too much about seeing a natural log or exponentials. Immediately set your sights on where the variables are defined and determine what shape is described by them,

especially when they are added or subtracted together. From there, it should be like a pre-calculus problem in terms of graphing equations.

Drawing Contour Lines

Refer to the following [video](#) by Andrew Bulawa on YouTube for understanding better how to do level curves if my explanation didn't properly suffice (considering I did not have time to make a video myself).

Limits of Variables

With regards to limits, you may think that this is a bit daunting because limits were something we focused on a lot in Calc 1 but not as much later. However, don't worry yourself about it. For me, it helps to take each variable one at a time to zero and see if it is defined and the same. Then I will define another line (one that reduces the function with two variables into one or something reduce if it is three variables). To explore this more refer to the following videos:

- [Organic Chemistry Tutor](#)
- [Prime Newtons](#)

Partial Derivatives

When approaching partial derivatives, my suggestion is to focus on the variable in question and take the derivative as if the other variable(s) is not considered. Afterward, add in the missing variables only if they were multiplied to the variable, we took the derivative for. If it was added in a term by itself or other variables, then no need to add it into the equation. For more problems outside of class refer to the following:

- [Organic Chemistry Tutor](#) (problems)
- [Professor Leonard](#) (more of explanation of concepts):

Tangent Planes and Linear Approximations

This is very similar to tangent lines and linear approximations we did in Calculus I. I would say approach these problems as such. Yes, there may be more variables involved. However, the fundamental concepts are very similar. By following the equations for tangent planes and linear approximations, you should be fine for this section. If you need another way of approaching the textbook and rehashing concepts as well as problems using the following videos:

- https://www.youtube.com/watch?v=rmf_vnM3ZlW
- <https://www.youtube.com/watch?v=lOOnZYZFQ18>

The following video is another individual explaining concepts and going through different problems if needed:

- <https://www.youtube.com/watch?v=yLbqHfuWsr8>

Chain Rule

Likewise, the chain rule is like the chain rule we have done before in the past. I like to think of crossing out the numerator and denominator for chain rule with multiple variables to get the desired term (although that's technically not what's happening here). I have linked videos that further explains the chain rule and goes over many problems.

- [Organic Chemistry Tutor](#)
- [Professor Leonard](#)
- [Krista King](#)

Directional Derivatives and Gradient Vector

For this section, I would suggest doing many practice problems and make sure you write down definitions for the directional derivatives and gradient vectors. One you go through the motions again and remember subsequent steps, these problems should be easier. Don't forget though to be able to define these as that tends to be a question that comes up. Use the following videos to help:

- [Professor Leonard](#) (Explanation on topics and problems)
- [Organic Chemistry Tutor](#) (problems)
- [Krista King](#) (problem):
- [James Hamblin](#) (problem)

Maximum and Minimum Values

Just as we used to find the maximum and minimum values in Calculus 1, this is very similar. Second Derivative Test will be your friend here and even takes a similar form to when we used it in Calculus 1. For a refresher and some practice problems use the following video links:

- Organic Chemistry Tutor:
 - <https://www.youtube.com/watch?v=UID893EosM8&pp=ygUqbWF4aW11bSBhb mQgbWluaW11bSBtdWx0aXZhcmlhYmxlIGNhbnGN1bHVz>
 - <https://www.youtube.com/watch?v=Hg38kfK5w4E&pp=ygUqbWF4aW11bSBhb mQgbWluaW11bSBtdWx0aXZhcmlhYmxlIGNhbnGN1bHVz>
- [Krista King](#)

Lagrange Multipliers

This is another topic that I believe it takes getting used to by many practice problems. I think it's important to remember what the Lagrange multipliers are supposed be used for.

Lagrange multipliers are supposed be used to find local maxima and minima of functions when constraints are present. Therefore, if a problem is set up for you to find local maxima or minima

and constraints, immediately think of this technique as opposed to second derivatives and saddles points as used in the previous section.

For a refresher on the topic and practice outside of what has been given, using the following videos:

- [Professor Leonard](#)
- [Organic Chemistry Tutor](#)